International Journal of Theoretical Physics, Vol. 39, No. 8, 2000

Symmetries and Conservation Laws in Theories with Higher Derivatives

I. Damian[1]

Received March 17, 2000

The general variation of the action depending on derivatives of an arbitrary order and two classes of variables is performed. The conservation of some generalized magnitudes associated to space-time and internal symmetries are studied.

1. INTRODUCTION

Theories with high-order derivatives have been used in different fields of physics. They are used for nonlocal theories and offer a realistic method of regularization. The introduction of high-order derivatives has also been proposed in supersymmetric theories [1, 6, 7]. In SUSY theories auxiliary independent variables have been introduced and the Schwinger action principle generalized to include such variables [4]. In ref. 3 some symmetries in theories in which the Lagrangian contains two classes of independent variables and derivatives up to the second order have been examined. In the present paper we extend the study of the symmetries for the case when higher order derivatives are present in a Lagrangian with two classes of independent variables.

We define the action by

$$A = \int d\theta \, dx \, L(x_\lambda, \theta_\alpha, \varphi_k, \partial^n_{\lambda\ldots}\varphi_k, \partial^r_{\alpha\ldots}\varphi_k) \tag{1.1}$$

in which L is the Lagrangian density, x_λ are space-time variables, θ_α are the suppplementary independent Grassmann variables, and φ_k are the generalized coordinates (fields). The following notations have been used:

[1] Universitatea “Politehnica”, Catedra de Fizica, Piata Regina Maria 1, 1900, Timisoara, Romania; e-mail: idamian@etv.utt.ro

0020-7748/00/0800-2141$18.00/0

$$
\begin{aligned}
dx &= \prod_{\lambda} dx_{\lambda} & d\theta &= \prod_{\alpha} d\theta_{\alpha} \\
\partial^{n}_{\lambda\ldots} &= \partial^{n}/\partial x_{\lambda}\partial x_{\mu}\ldots & \partial^{r}_{\alpha\ldots} &= \partial^{r}/\partial\theta_{\alpha}\,\partial\theta_{\beta}\ldots \\
n &= 1, 2, \ldots, Z_n & r &= 1, 2, \ldots, Z_r
\end{aligned}
\tag{1.2}
$$

in which Z_n and Z_r are the highest order of derivatives. For the sake of simplicity we have assumed that L contains derivatives of the same maximum order of all appearing functions in a class of variables.

2. THE ACTION PRINCIPLE AND FIELD EQUATIONS

The action principle with fixed boundaries for all integrals reads

$$\delta_0 A = \delta_0 \int d\theta\, dx\, L = \int d\theta \int dx\, \delta_0 L = 0 \tag{2.1}$$

where the variation $\delta_0 L$ is

$$
\begin{aligned}
\delta_0 L = {} & L(\varphi_k + \delta_0\varphi_k,\ \partial^{n}_{\lambda\ldots}\varphi_k + \delta_0\partial^{n}_{\lambda\ldots}\varphi_k,\ \partial^{r}_{\alpha\ldots}\varphi_k + \delta_0\partial^{r}_{\alpha\ldots}\varphi_k) \\
& - L(\varphi_k,\ \partial^{n}_{\lambda\ldots}\varphi_k,\ \partial^{r}_{\alpha\ldots}\varphi_k)
\end{aligned}
\tag{2.2}
$$

Applying a Taylor expansion to the first term on the right-hand side in (2.2) and keeping only first order terms, we obtain

$$
\begin{aligned}
\delta_0 L = {} & \sum_{k} \frac{\partial L}{\partial\varphi_k}\,\delta_0\varphi_k + \sum_{k}\sum_{n}\sum_{(\lambda)} \frac{\partial L}{\partial(\partial^{n}_{\lambda\ldots}\varphi_k)}\,\delta_0\partial^{n}_{\lambda\ldots}\varphi_k \\
& + \sum_{k}\sum_{r}\sum_{(\alpha)} \frac{\partial L}{\partial(\partial^{r}_{\alpha\ldots}\varphi_k)}\,\delta_0\partial^{r}_{\alpha\ldots}\varphi_k
\end{aligned}
\tag{2.3}
$$

with short-hand notations

$$\sum_{(\lambda)} = \sum_{\lambda}\sum_{\mu}\ldots, \qquad \sum_{(\alpha)} = \sum_{\alpha}\sum_{\beta}\ldots \tag{2.4}$$

In the sums we retain identical terms only once.

After lengthy calculation we obtain

$$
\begin{aligned}
\delta_0 A = \int d\theta \int dx \Bigg\{ & \frac{\partial L}{\partial\varphi_k}\,\delta_0\varphi_k + \sum_{r}\sum_{(\alpha)} (-\,1)^{r}\left(\partial^{r}_{\alpha\ldots}\,\frac{\partial L}{\partial(\partial^{r}_{\alpha\ldots}\varphi_k)}\right)\delta_0\varphi_k \\
& + \sum_{\eta} \partial_{\eta}\left[\sum_{r}\sum_{f=0}^{r-1}\left(\sum_{(\alpha)}\nolimits_{|\eta|} (-1)^{j}\,\partial^{f}_{\alpha\ldots[\eta-1]}\,\frac{\partial L}{\partial(\partial^{r}_{\alpha\ldots\eta\beta\ldots}\varphi_k)}\right)\partial^{r-f-1}_{[\eta+1]}\delta_0\varphi_k\right] \\
& + \sum_{n}\sum_{(\lambda)} (-1)^{n}\left(\partial^{n}_{\lambda\ldots}\,\frac{\partial L}{\partial(\partial^{n}_{\lambda\ldots}\varphi_k)}\right)\delta_0\varphi_k
\end{aligned}
$$

$$+ \sum_{\rho} \partial_{\rho}\left[\sum_{n} \sum_{g=0}^{n-1} \left(\sum_{(\lambda)}{}_{|\rho|} (-1)^{g}\, \partial^{g}_{\lambda\ldots[\rho-1]} \right.\right.$$

$$\left.\left. \times \frac{\partial L}{\partial(\partial^{n}_{\lambda\ldots\rho\mu\ldots}\varphi_{k})}\right) \partial^{n-g-1}_{[\rho+1]}\, \delta_{0}\varphi_{k}\right]\Bigg\} = 0 \tag{2.5}$$

in which $\eta = [\alpha + f]$ and $\rho = [\alpha + g]$ and the subscripts $|\eta|$ and $|\rho|$ indicate that the sums over these indices are missing. Summation over k was omitted and it will be understood from now on.

One can see that the terms 3 and 5 vanish by integration because the boundaries of the integrals are fixed, and since the variations $\delta_0\varphi_k$ are arbitrary the remaining terms yield

$$\frac{\partial L}{\partial\varphi_{k}} + \sum_{r}\sum_{(\alpha)} (-1)^{r}\, \partial^{r}_{\alpha\ldots} \frac{\partial L}{\partial(\partial^{r}_{\alpha\ldots}\varphi_{k})} + \sum_{n}\sum_{(\lambda)} (-1)^{n}\, \partial^{r}_{\lambda\ldots} \frac{\partial L}{\partial(\partial^{n}_{\lambda\ldots}\varphi_{k})} = 0 \tag{2.6}$$

Considering the θ_a as Grassmann variables, we can interpret Eq. (2.6) as the field equations in superspace with higher derivatives. If the highest order of derivatives is 2, there result the equations of field obtained by Borneas and Damian [3].

3. GENERAL VARIATION OF THE ACTION

The general variation of the action, including variation of the boundaries of the integrals over x_λ, is

$$\delta A = \delta \int d\theta \int dx\, L = \int d\theta\, \delta \int dx\, L \tag{3.1}$$

where

$$\delta \int dx\, L = \int (\delta\, dx) L + \int dx\, \delta L \tag{3.2}$$

and

$$\int (\delta\, dx) L = \int dx \sum_{\lambda} (\partial_{\lambda}\, \delta x_{\lambda}) L = \int dx \left[\sum_{\lambda} \partial_{\lambda}(L\, \delta x_{\lambda}) - \sum_{\lambda} (\partial_{\lambda} L)\, \delta x_{\lambda}\right] \tag{3.3}$$

The variation of L is given by

$$\delta L = \frac{\partial L}{\partial \varphi_k}\,\delta\varphi_k + \sum_r \sum_{(\alpha)} \frac{\partial L}{\partial(\partial^r_{\alpha\ldots}\varphi_k)}\,\delta\partial^r_{\alpha\ldots}\varphi_k + \sum_n \sum_{(\lambda)} \frac{\partial L}{\partial(\partial^n_{\lambda\ldots}\varphi_k)}\,\delta\partial^n_{\lambda\ldots}\varphi_k \tag{3.4}$$

in which the increments are

$$\delta\varphi_k = \delta_0\varphi_k + \sum_\alpha (\partial_\alpha\varphi_k)\,\delta\theta_\alpha + \sum_\lambda (\partial_\lambda\varphi_k)\,\delta x_\lambda \tag{3.5}$$

$$\delta\partial^r_{\alpha\ldots}\varphi_k = \delta_0\partial^r_{\alpha\ldots}\varphi_k + \sum_\varepsilon (\partial_\varepsilon\partial^r_{\alpha\ldots}\varphi_k)\,\delta\theta_\varepsilon + \sum_\pi (\partial_\pi\partial^r_{\alpha\ldots}\varphi_k)\,\delta x_\pi \tag{3.6}$$

$$\delta\partial^n_{\lambda\ldots}\varphi_k = \delta_0\partial^n_{\lambda\ldots}\varphi_k + \sum_\varepsilon (\partial_\varepsilon\partial^n_{\lambda\ldots}\varphi_k)\,\delta\theta_\varepsilon + \sum_\pi (\partial_\pi\partial^n_{\lambda\ldots}\varphi_k)\,\delta x_\pi \tag{3.7}$$

Taking into account (3.2)–(3.7), after a long calculation, we can put the variation (3.1) in the form

$$\begin{aligned}\delta A = \int d\theta \int dx \sum_\rho \partial_\rho\Bigg[&\sum_{h=1}^{Z_n}\sum_{f=0}^{Z_n-1} B^{k\rho}_{hf}\,\delta\partial^{h-1}_{[\rho+1]\ldots}\varphi_k \\ &- \sum_\pi\left(\sum_{h=1}^{Z_n}\sum_{f=0}^{Z_n-1} B^{k\rho}_{hf}\,\partial_\pi\partial^{h-1}_{[\rho+1]\ldots}\varphi_k - L\delta^{\pi\rho}\right)\delta x_\pi \\ &- \sum_\pi\left(\sum_{h=1}^{Z_n}\sum_{f=0}^{Z_n-1} B^{k\rho}_{hf}\,\partial_\varepsilon\,\partial^{h-1}_{[\rho+1]\ldots}\varphi_k\right)\delta\theta_g\end{aligned} \tag{3.8}$$

where

$$B^{k\rho}_{hf} = \sum_{(\lambda)}{}_{|\rho|}\,(-1)^f\,\partial^f_{\lambda\ldots[\rho-1]}\,\frac{\partial L}{\partial(\partial^{f+h}_{\lambda\ldots\rho\mu\ldots}\varphi_k)} \tag{3.9}$$

and $\delta^{\pi\rho}$ is the Kronecker symbol.

4. SPACE-TIME INVARIANCE

We study now the invariance of the action under different transformations. First consider space-time infinitesimal transformations. In this case only transformations δx_π occur in (3.8) and the invariance of the action with respect to these transformations reads

$$\delta A = \int d\theta \int dx \sum_\rho \partial_\rho(-\mathcal{T}^{\pi\rho}\delta x_\pi) = 0 \tag{4.1}$$

where we have denoted

$$\mathcal{T}^{\pi\rho} = \sum_{h=1}^{Z_n} \sum_{f=0}^{Z_n-1} B_{hf}^{k\rho}\ \partial_\pi \partial_{[\rho+1]..}^{h-1} \varphi_k - L\delta^{\pi\rho} \tag{4.2}$$

With δx_π independent of x_π, (4.1) can be written in the form

$$\delta A = \int d\theta \int dx \sum_\pi \left[-\left(\sum_\rho \partial_\rho \mathcal{T}^{\pi\rho} \right) \delta x_\pi \right] = 0 \tag{4.3}$$

which, for arbitrary variation of δx_π, leads to

$$\sum_\rho \partial_\rho \mathcal{T}^{\pi\rho} = 0 \tag{4.4}$$

The relation (4.4) indicates the conservation of the magnitude $\mathcal{T}^{\pi\rho}$, which we call the generalized energy tensor in the theory with high-order derivatives. If one neglects the dependence of the Lagrangian on derivatives whose order is higher than 1, one obtains the energy tensor from usual field theory,

$$T^{\pi\rho} = \frac{\partial L}{\partial(\partial_\rho \varphi_k)}\ \partial_\pi \varphi_k - L\delta^{\pi\rho} \tag{4.5}$$

which satisfies a conservation law in the form (4.4).

Let us now write the elementary variation of the independent variables by the linear relations

$$\delta x_\pi = \sum_\sigma e_{\pi\sigma} x_\sigma + e_\pi \tag{4.6}$$

with

$$e_{\pi\sigma} + e_{\sigma\pi} = 0 \tag{4.7}$$

where e are first-order infinitesimals. Inserting (4.6) in (4.1), one obtains

$$\begin{aligned}
\delta A &= \int d\theta \int dx \sum_\rho \partial_\rho \left\{ -\sum_\pi \left[\mathcal{T}^{\pi\rho} \left(\sum_\sigma e_{\pi\sigma} x_\sigma + e_\pi \right) \right] \right\} \\
&= \int d\theta \int dx \sum_\rho \partial_\rho \left[-\sum_\pi \sum_\sigma \mathcal{T}^{\pi\rho} e_{\pi\sigma} x_\sigma - \sum_\pi \mathcal{T}^{\pi\rho} e_\pi \right] \\
&= \int d\theta \int dx \sum_\rho \partial_\rho \left[-\frac{1}{2} \sum_\pi \sum_\sigma (\mathcal{T}^{\pi\rho} e_{\pi\sigma} x_\sigma + \mathcal{T}^{\sigma\rho} e_{\sigma\pi} x_\pi) - \sum_\pi \mathcal{T}^{\pi\rho} e_\pi \right] \\
&= \int d\theta \int dx \sum_\rho \partial_\rho \left[-\frac{1}{2} \sum_\pi \sum_\sigma (\mathcal{T}^{\pi\rho} x_\sigma - \mathcal{T}^{\sigma\rho} x_\pi) e_{\pi\sigma} - \sum_\pi \mathcal{T}^{\pi\rho} e_\pi \right]
\end{aligned}$$

$$= \int d\theta \int dx \left\{ -\frac{1}{2} \sum_{\pi} \sum_{\sigma} \left[\sum_{\rho} \partial_\rho (\mathcal{T}^{\pi\rho} x_\sigma - \mathcal{T}^{\sigma\rho} x_\pi) \right] e_{\pi\sigma} \right.$$

$$\left. - \sum_{\pi} \left(\sum_{\rho} \partial_\rho \mathcal{T}^{\pi\rho} \right) e_\pi \right\} = 0 \tag{4.8}$$

If only transformations e_π occur, this leads to the conversation law (4.4). If separate transformations $e_{\pi\sigma}$ occur in (3.8), then one obtains the conservation of the magnitude

$$\mathcal{R}^{\pi\sigma\rho} = \mathcal{T}^{\pi\rho} x_\sigma - \mathcal{T}^{\sigma\rho} x_\pi \tag{4.9}$$

We observe that in the absence of high-order derivatives, $\mathcal{R}^{\pi\sigma\rho}$ reduces to the "orbital" angular momentum tensor of the usual field theory, so we are justified to consider the magnitude (4.9) as the generalized "orbital" angular momentum tensor of the present theory.

If only $\delta\theta_\alpha$ transformations occur, the invariance of the action leads to the conservation of the magnitude

$$O^{\pi\rho} = \sum_{h=1}^{Z_n} \sum_{f=0}^{Z_n - 1} B_{hf}^{kp} \partial_\varepsilon\, \partial_{[\rho+1]\ldots}^{h-1} \varphi_k \tag{4.10}$$

which can eventually be interpreted in the frame of a supersymmetric theory which considers auxiliary variables.

5. INTERNAL SYMMETRY IN CLASSICAL FIELDS

Let us consider a classical complex field φ_k and an infinitesimal phase transformation of the form

$$\begin{aligned} \varphi_k' &= \varphi_k + i\varepsilon\varphi_k \\ \varphi_k'^{*} &= \varphi_k^{*} - i\varepsilon\varphi_k^{*} \end{aligned} \tag{5.1}$$

where ε is a real, arbitrary constant. So we also have

$$\begin{aligned} \delta\varphi_k &= i\varepsilon\varphi_k \\ \delta\varphi_k^{*} &= -i\varepsilon\varphi_k^{*} \end{aligned} \tag{5.2}$$

and

$$\begin{aligned} \partial_{\lambda\ldots}^{n} \delta\varphi_k &= i\varepsilon \partial_{\lambda\ldots}^{n} \varphi_k \\ \partial_{\lambda\ldots}^{n} \delta\varphi_k^{*} &= -i\varepsilon \partial_{\lambda\ldots}^{n} \varphi_k^{*} \end{aligned} \tag{5.3}$$

We require the action to be invariant under the above transformations, so, with $\delta x_\lambda = \delta\theta_\alpha = 0$, we have from (3.8) and (3.9)

$$\partial_\rho \Bigg[\sum_{h=1}^{Z_n} \sum_{f=0}^{Z_n-1} \sum_{(\lambda)}{}_{|\rho|} (-1)^f \, \partial^f_{\lambda\ldots[\rho-1]} \frac{\partial L}{\partial(\partial^{f+h}_{\lambda\ldots\rho\mu\ldots}\varphi_k)} \, \delta\partial^{h-1}_{[\rho+1]\ldots}\varphi_k + \sum_{h=1}^{Z_n} \sum_{f=0}^{Z_n-1} \sum_{(\lambda)}{}_{|\rho|} (-1)^f \, \partial^f_{\lambda\ldots[\rho-1]} \frac{\partial L}{\partial(\partial^{f+h}_{\lambda\ldots\rho\mu\ldots}\varphi^*_k)} \, \delta\partial^{h-1}_{[\rho+1]\ldots}\varphi^*_k \Bigg] = 0 \tag{5.4}$$

Introducing (5.2) and (5.3) in (5.4), we obtain

$$\partial_\rho I_\rho = 0 \tag{5.5}$$

indicating the conservation of the magnitude

$$I_\rho = i\varepsilon \Bigg[\sum_{h=1}^{Z_n} \sum_{f=0}^{Z_n-1} \sum_{(\lambda)}{}_{|\rho|} (-1)^f \, \partial^f_{\lambda\ldots[\rho-1]} \frac{\partial L}{\partial(\partial^{f+h}_{\lambda\ldots\rho\mu\ldots}\varphi_k)} \, \partial^{h-1}_{[\rho+1]\ldots}\varphi_k + \sum_{h=1}^{Z_n} \sum_{f=0}^{Z_n-1} \sum_{(\lambda)}{}_{|\rho|} (-1)^f \, \partial^f_{\lambda\ldots[\rho-1]} \frac{\partial L}{\partial(\partial^{f+h}_{\lambda\ldots\rho\mu\ldots}\varphi^*_k)} \, \partial^{h-1}_{[\rho+1]\ldots}\varphi^*_k \Bigg] \tag{5.6}$$

which we interpret as the generalized four-vector current in the present theory. If the highest order of derivatives in the Lagrangian is 2, one obtains the result from Borneas and Damian [3], and in the absence of higher derivatives all the supplementary terms cancel and I_ρ reduces to the usual four-vector current.

The quantity

$$Q = -i \int dV \, I_4 \tag{5.7}$$

is the total charge. For real fields, $\varphi_k = \varphi^*_k$, and thus real fields are neutral.

6. INTERNAL SYMMETRY IN QUANTUM FIELDS

The infinitesimal unitary transformation of a quantum field φ_k is given by

$$\begin{aligned} \varphi_k \to \varphi'_k &= (1 + i\varepsilon\Lambda_r G_r)\varphi_k(1 + i\varepsilon\Lambda_r G_r) \\ &= \varphi_k + i\varepsilon\Lambda_r[G_r, \varphi_k] \end{aligned} \tag{6.1}$$

where G_r are generators of a Lie group.

Denoting

$$[G_r, \varphi_k] = (M_r)_{kl}\varphi_l \tag{6.2}$$

one can write

$$\delta\varphi_k = i\varepsilon\Lambda_r(M_r)_{kl}\varphi_k \tag{6.3}$$

The Lagrangian must be invariant under this transformation; therefore we write

$$\delta L = \frac{\partial L}{\partial \Lambda_r}\,\delta\Lambda_r = \frac{\partial L}{\partial \varphi_k}\frac{\delta\varphi_k}{\delta\Lambda_r}\,\delta\Lambda_r + \sum_n \sum_{(\lambda)} \frac{\partial L}{\partial(\partial^n_{\lambda\ldots}\varphi_k)}\frac{\delta(\partial^n_{\lambda\ldots}\varphi_k)}{\delta\Lambda_r}\,\delta\Lambda_r = 0 \tag{6.4}$$

But we have

$$\frac{\delta\varphi_k}{\delta\Lambda_r} = i\varepsilon(M_r)_{kl}\varphi_l \tag{6.5}$$

$$\frac{\delta(\partial^n_{\lambda\ldots}\varphi_k)}{\delta\Lambda_r} = \partial^n_{\lambda\ldots}\frac{\delta\varphi_k}{\delta\Lambda_r} = \partial^n_{\lambda\ldots}[i\varepsilon(M_r)_{kl}\varphi_l] \tag{6.6}$$

Introducing (6.5) and (6.6) in (6.4), one obtains

$$\delta L = \frac{\partial L}{\partial \varphi_k}[i\varepsilon(M_r)_{kl}\varphi_l] + \sum_n \sum_{(\lambda)} \frac{\partial L}{\partial(\partial^n_{\lambda\ldots}\varphi_k)}\,\partial^n_{\lambda\ldots}[i\varepsilon(M_r)_{kl}\varphi_l] = 0 \tag{6.7}$$

Applying successively the general relation [2]

$$\begin{aligned}(-1)^f&\left(\partial^f_{\lambda\ldots[\lambda+f-1]}\frac{\partial L}{\partial(\partial^n_{\lambda\ldots}\varphi_k)}\right)\partial^{n-f}_{[\lambda+f]\ldots}[i\varepsilon(M_r)_{kl}\varphi_l]\\ &= (-1)^f\,\partial_{[\lambda+f]}\left[\left(\partial^f_{\lambda\ldots[\lambda+f-1]}\frac{\partial L}{\partial(\partial^n_{\lambda\ldots}\varphi_k)}\right)\partial^{n-f-1}_{[\lambda+f+1]\ldots}[i\varepsilon(M_r)_{kl}\varphi_l]\right]\\ &\quad - (-1)^f\left(\partial^{f+1}_{\lambda\ldots[\lambda+f]}\frac{\partial L}{\partial(\partial^n_{\lambda\ldots}\varphi_k)}\right)\partial^{n-f-1}_{[\lambda+f+1]\ldots}[i\varepsilon(M_r)_{kl}\varphi_l]\end{aligned} \tag{6.8}$$

to the second term of (6.7), one obtains

$$\begin{aligned}\delta L &= \frac{\partial L}{\partial \varphi_k}[i\varepsilon(M_r)_{kl}\varphi_l] + \sum_n \sum_{(\lambda)} (-1)^n\left(\partial^n_{\lambda\ldots}\frac{\partial L}{\partial(\partial^n_{\lambda\ldots}\varphi_k)}\right)[i\varepsilon(M_r)_{kl}\varphi_l]\\ &\quad + \sum_n \sum_{(\lambda)} \sum_{f=0}^{n-1} (-1)^f\,\partial_{[\lambda+f]}\left[\left(\partial^f_{\lambda\ldots[\lambda+f-1]}\frac{\partial L}{\partial(\partial^n_{\lambda\ldots}\varphi_k)}\right)\right.\\ &\quad \left.\times\,\partial^{n-f-1}_{[\lambda+f+1]\ldots}[i\varepsilon(M_r)_{kl}\varphi_l]\right] = 0\end{aligned} \tag{6.9}$$

We rearrange now the last term in (6.9), taking the derivative $\partial_{[\lambda+f]}$ outside the sums, denoting $\lambda + f = \rho$ and $n - f = h$, and limiting the summation over f up to $Z_n - h$. In view of (2.6) for this case, (6.9) yields

$$\partial_\rho \left\{ \sum_{h=1}^{Z_n} \sum_{(\lambda)} \sum_{j=0}^{Z_n-h} (-1)^f \left[\left(\partial^f_{\lambda\ldots[\rho-1]} \frac{\partial L}{\partial(\partial^{f+h}_{\lambda\ldots}\varphi_k)} \right) \partial^{h-1}_{[\rho+1]\ldots}[i\varepsilon(M_r)_{kl}\varphi_l] \right] \right\} = 0 \qquad (6.10)$$

We interpret the quantity

$$I'_\rho = \sum_{h=1}^{Z_n} \sum_{(\lambda)} \sum_{f=0}^{Z_n-h} (-1)^f \left[\left(\partial^f_{\lambda\ldots[\rho-1]} \frac{\partial L}{\partial(\partial^{f+h}_{\lambda\ldots}\varphi_k)} \right) \partial^{h-1}_{[\rho+1]\ldots}[i\varepsilon(M_r)_{kl}\varphi_l] \right] \qquad (6.11)$$

as the generalized four-vector current and (6.10) represent its conservation. As one can see, I'_p is made up of the usual four-vector current and supplementary terms due to including of high-order derivatives. Of course, as in the classical case, there is a total charge containing a supplementary charge added to the usual one, and the total charge is conserving, not the usual one.

7. CONCLUSIONS

The question of the absoluteness of many conservation laws has often been discussed (for instance, Mohapatra [5] dealing with charge conservation). Our approach has regarded some conservaton laws in the frame of a theory with high-order derivatives and extends the results obtained in a previous paper [3]. It results in the conservation of some generalized magnitudes which are more complex than the corresponding classical ones. For instance, the total (generalized) charge is conserved, but the usual charge is only conserved in some cases, when the supplementary terms due to the presence of higher derivatives in the Lagrangian are negligible.

REFERENCES

1. A. Andrianov, M. V. Ioffe, and V. P. Spiridonov, (1993) *Phys. Lett. A* **174**, 273.
2. M. Borneas, U.T.F.T. (I), nr. 8/88; U.T.F.T. (II), nr. 9/88, Univ. Timisoara, preprints.
3. M. Borneas and I. Damian, (1999) *Int. J. Theor. Phys.* **38**, 2239.
4. S. P. Misra, (1983) *Phys. Lett.* **129B**, 401.
5. R. N. Mohapatra, (1987) *Phys. Rev. Lett.* **59**, 1510.
6. M. A. Namazie, (1980) *J. Phys. A* **11**, 713.
7. O. Yasuda, (1986) *Phys. Lett.* **169B**, 64.